# The Effects of Capital Requirements on Good and Bad Risk Taking[*]

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### Abstract

We identify a new channel by which deposit insurance and capital requirements affect welfare in general equilibrium. In the model, entrepreneurs have access to socially valuable projects whose return is subject to idiosyncratic, uninsurable risk. This risk reduces the entrepreneurs' investment and gives rise to a demand for safe assets, which are supplied by banks. If the government provides subsidized deposit insurance, banks pay a higher risk-adjusted return on deposits and entrepreneurs increase their investment in the socially-valuable projects. The optimal capital requirement trades off this effect with the moral hazard induced by deposit insurance.

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# 1  Introduction

After the 2007–2008 financial crisis, the revision of the regulatory framework of financial intermediaries has become a central topic under discussion by regulators and academics. The Basel III accord has tightened the restrictions imposed on banks by previous regulation and has introduced new tools to reduce the likelihood and depth of financial crises. One of the key sets of rules at the center of this debate is capital requirements, namely, limits on the fraction of debt that banks can use to finance their investment.

According to the Modigliani and Miller (1958) theorem, it is irrelevant whether a firm is financed with debt, equity, or a mix of the two. Admati and Hellwig (2013) use this result as a starting point for their analysis of bank regulation. They argue that capital requirements should be raised substantially to eliminate the moral hazard induced by government guarantees and the implicit too-big-to-fail subsidies, and some regulators have made a case for similar rules.[1] The typical argument against this suggestion is that banks are "special" because their liabilities are valued not only for their pecuniary return but also for their "liquidity'' value, violating a key Modigliani-Miller assumption. Under this view, it is not desirable to impose large capital requirements that reduce the supply of banks deposits.

In this paper, we propose an alternative channel that reduces the desirability of high capital requirements. This channel is related to the role of banks as suppliers of safe assets and is different—though complementary—to the typical transaction role of deposits. In our model, the demand for safe assets comes from entrepreneurs who have access to projects that are socially valuable but whose return is subject to idiosyncratic, uninsurable risk. It is this idiosyncratic risk that gives raise to a demand for safe assets—or, more precisely, for assets whose return does not correlate with the entrepreneurs' projects.

A crucial result of our model is that the return on safe assets interacts with entrepreneurs' willingness to pursue risky, socially-valuable projects. The idiosyncratic shocks to the return on entrepreneurs' projects create volatility in entrepreneurs' own wealth. Because of this

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[1]See the Minneapolis Plan discussed by Kashkari (2016).

volatility, entrepreneurs reduce their investment in the projects in comparison to an economy without idiosyncratic shocks. In this context, a higher return on safe assets provides a source of stable income that reduces the volatility of entrepreneurs' wealth. This effect encourages entrepreneurs to take on more risk, increasing their demand for labor and the scale of their idiosyncratic projects. Vice-versa, a lower return on safe assets reduces entrepreneurial activity. We call this channel "good risk taking" because the projects are socially valuable but not fully exploited due to the lack of insurance against idiosyncratic risk.

We think of the entrepreneurs in our model as small, privately-owned firms. Such firms are not included in standard datasets, such as Compustat, because they do not issue equity or file with the SEC; as such, they are under-represented in the finance literature relative to publicly-traded firms. However, privately-owned firms account for more than half of total domestic non-government employment in the US.[2] Thus any force affecting employment in this sector can have large effects on aggregate output and employment.

We highlight a new cost of capital requirements: they reduce the good risk taking of entrepreneurs. If the government provides subsidized deposit insurance, the subsidy increases the risk-adjusted return on deposits. Since deposits are safe assets demanded by entrepreneurs for insurance purposes, the higher return promotes good risk taking by entrepreneurs according to the channel explained above. As a result, capital requirements that limit deposit insurance disbursements and reduce the return on deposits also reduce good risk taking, and produce a negative impact on growth and welfare. The optimal level of capital requirements trades off this effect with the benefit of reducing the "bad" risk taking of banks, that is, the moral hazard of deposit insurance.

Our general message is that financial regulation has an impact on the real economy by changing the risk-taking capacity of the non-financial sector, and that this channel should be accounted for when studying optimal regulation. Because of this motivation, we focus on deposit insurance and capital requirements due to their primary role in banking regulation,

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[2]We arrive at this figure by dividing total employment at firms in the Compustat database by total private employment from the BLS each year and taking the average from 1950–2016.

and we abstract from other policies that might enhance entrepreneurs' risk-taking but are unrelated to the financial sector. In this sense, we follow a common approach in the literature that studies capital requirements and motivates deposit insurance because of its role in preventing runs, as in Diamond and Dybvig (1983), but does not model explicitly this feature. More generally, deposits insurance in our model can also be interpreted as any government guarantee on bank debt, such as the Temporary Liquidity Guarantee Program set up by the Federal Deposit Insurance Corporation (FDIC) in 2008.

We first derive our results using some simplifying assumptions that keep our model tractable and allow us to isolate our channel from other effects. In particular, even if idiosyncratic shocks create heterogeneity across entrepreneurs' wealth, the equilibrium in our model depends only on average wealth, and the other moments of the distribution are irrelevant. We also assume that the government finances the shortages of the deposit insurance funds with taxes that do not create any distortions in entrepreneurs' decisions. We then plan to extend the paper by providing a quantitative analysis and assessing the magnitude of the good risk-taking channel.

## 2    Literature Review

This paper is part of a growing literature that studies capital requirements using macroeconomic models with a financial sector. Several papers employ quantitative models, such as Begenau (2016), Begenau and Landvoigt (2017), Corbae and D'Erasmo (2014), Christiano and Ikeda (2013), Davydiuk (2017), Dempsey (2017), Gertler, Kiyotaki and Prestipino (2016), Nguyễn (2014), and van den Heuvel (2008).

A related paper by Elenev, Landvoigt and van Nieuwerburgh (2017) studies the effect of increasing the price of mortgage guarantee offered by government-sponsored enterprises (GSEs). While they focus on a different policy, we share with them the idea that regulation and subsidies to the financial sector can have an impact on wealth distribution.

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Our approach for modeling entrepreneurs' risk builds on Quadrini (2017), who also emphasizes the role of bank liabilities for insurance purposes.[3] However, his focus is different because he studies how various forces affect banks' risk taking and crises, without focusing on entrepreneurs' risk taking.

Our paper is also related to the literature that studies financial intermediaries as suppliers of safe assets, such as Diamond (2016), Magill, Quinzii and Rochet (2016), and Stein (2012). This literature builds on the ideas of Dang et al. (2017) and Gorton and Pennacchi (1990), in which the debt of banks is riskless to enhance its liquidity value or to overcome an informational friction. Bank debt is valuable in our model for a related but slightly different reason: there is a demand for securities that are uncorrelated with the idiosyncratic risk of entrepreneurs. Policies that increase the increase the supply of such securities also increase entrepreneurs' risk-taking capacity.

# 3 Model

## 3.1 Environment

Time is discrete and infinite and there is a single good that can be consumed or used for investment. There are four types of players in the economy: agents (entrepreneurs), banks, laborers, and the government.

### 3.1.1 Agents

Agents have log utility and discount the future at a rate $\beta < 1$. Agents live forever and and choose consumption to maximize their expected discounted stream of utility:

$$U^i = E \sum_{t=0}^{\infty} \beta^t \log c_t^i. \tag{1}$$

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[3]The approach used by Quadrini (2017) at modeling entrepreneurs' risk builds on Arellano, Bai and Kehoe (2011).

Agents hire workers and produce output equal to $z^i_{t+1} l^i_t$, where $z^i_{t+1}$ is the firm's productivity and $l^i_t$ is their labor input. $z^i_{t+1}$ is an idiosyncratic productivity shock that is realized after agents choose their labor input $l^i_t$. Agents can invest in deposits $d^i_t$ that pay an interest rate $R^d_t$, and in bank's equity $n^i_t$ that earns a (stochastic) return $R^E_{t+1}$; their budget equation is

$$c^i_t + d^i_t + n^i_t = x^i_t,$$

where $x^i_t$ is their total wealth at time $t$. Agent $i$'s wealth $x^i_t$ evolves as

$$x^i_{t+1} = (1 - \tau_{t+1}) \left[ \left( z^i_{t+1} - w_t \right) l^i_t + R^d_t d^i_t + R^E_{t+1} n^i_t \right]$$

where $\tau_{t+1}$ is a tax levied by the government to pay back depositors at failed banks.

Agents choose their investments in bonds, equity, and labor before knowing their own idiosyncratic productivity draw $z^i_{t+1}$ or the realization of the return on equity $R^E_{t+1}$. However, the government provides full deposit insurance, so deposits are safe and $R^d_t$ is known in advance. The wage $w_t$ is also known at time $t$ when $l^i_t$ is chosen.

A note on timing and notation: all variables indexed with a $t$ subscript are known to agents at the beginning of time $t$ when they make decisions. Thus for the agent the only unknowns are their own productivity $z^i_{t+1}$, the return on equity $R^E_{t+1}$, and the tax rate $\tau_{t+1}$. These random variables depend on future aggregate productivity $A_{t+1}$, described below. Agent's idiosyncratic output $z^i_{t+1} l^i_t$ depends on productivities and occurs at the beginning of period $t + 1$, immediately after the shocks are realized.

The above assumptions lead to the following Bellman equation for agent $i$:

$$V_t\left(x_t^i\right) = \max_{c_t^i, d_t^i, n_t^i, l_t^i \geq 0} \log c_t^i + \beta E_t V_{t+1}\left(x_{t+1}^i\right)$$

$$s.t.$$

$$c_t^i + d_t^i + n_t^i = x_t^i$$

$$x_{t+1}^i = (1 - \tau_{t+1})\left[\left(z_{t+1}^i - w_t\right)l_t + R_t^d d_t^i + R_{t+1}^E n_t^i\right]$$

where the $t$ subscript on the value function incorporate all aggregate information at time $t$, including the distribution of wealth $x_t^i$ across agents. As we show below in Proposition 1, the wealth distribution will not enter into agents' optimal decisions, a key result that keeps our model tractable. This result stems from our assumption of log utility, and from the fact that the agents' problem exhibits constant returns to scale.

The assumption that the government taxes agents in proportion to agents' wealth $x_t^i$, rather than lump-sum or on project income $\left(z_{t+1}^i - w_t\right)l_t^i$, allows us to isolate our channel from other effects. The combination of log utility and proportionality to wealth implies that agents' investment and labor-demand choices are independent of the level of the tax, as we show below, similar to the way lump-sum taxes work in other classes of models.[4] Moreover, a tax on project income would have a direct effect on agents' labor demand and would smooth the return on entrepreneurs' projects – the tax would be paid by entrepreneurs with successful projects, but not by those with unsuccessful projects. This channel would create a further effect that reduces entrepreneurs risk above and beyond that of deposit insurance and capital requirements. Because our goal is to understand the effects of capital requirements themselves, we have chosen to pay for deposit insurance in the model with a non-distortionary tax on wealth. We conjecture that a tax on income would lead to even stronger effects.

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[4]We do not use lump-sum taxes because they would make agents' decisions dependent on their own wealth, thereby eliminating the tractability advantages of log utility.

### 3.1.2 Banks

Banks live for a single period; that is, a bank set up at time $t$ is liquidated at the beginning of time $t + 1$. At the beginning of period $t$ they have total equity $n_t$ and have access to a decreasing-returns-to-scale technology which produces output equal to $A_{t+1}\varepsilon k_t^\alpha$, where $A_{t+1}$ is aggregate productivity, $\varepsilon \geq 0$ is a bank-specific idiosyncratic shock realized at $t + 1$, and $k_t$ is the physical capital invested in by the bank. $\varepsilon$ is an idiosyncratic shock, so we have that $E\{\varepsilon\} = 1$. Banks finance any investment in $k_t$ beyond their own equity capital $n_t$ by borrowing from agents at $R_t^d$. Banks face a capital requirement that limits their ability to raise deposits; formally, their equity ratio $n_t/k_t$ must be weakly larger than some number $\zeta$.

The banker's problem is

$$\max_{k_t, d_t} \ E_t \int \left\{ A_{t+1}\varepsilon k_t^\alpha - R_t^d d_t \right\}^+ dF(\varepsilon)$$

s.t.

$$k_t = d_t + n_t \tag{2}$$

$$\frac{n_t}{d_t + n_t} \geq \zeta$$

$$n_t \text{ given}$$

where the constraint in $\zeta$ reflects the capital requirement, $\{\cdot\}^+ = \max\{\cdot, 0\}$ is the positive part of bank's profits, $F(\cdot)$ is the CDF of banks' idiosyncratic productivity shocks, and the expectation is taken over the distribution of $A_{t+1}$. Banks that receive a low value of the productivity shock $\varepsilon$, such that $A_{t+1}e^{\sigma\varepsilon}k_t^\alpha < R_t^d d_t$, do not pay back their depositors; these deposits are guaranteed by the government.

When firms invest in bank's equity, they invest in a mutual fund that diversifies over the idiosyncratic shocks $\varepsilon$. Thus investments in bank equity are exposed to the aggregate risk in $A_{t+1}$ and to the fraction of banks that default in equilibrium. The realized return on equity

is given by

$$R_{t+1}^E = \frac{1}{n_t} \int \left\{ A_{t+1}\varepsilon k_t^\alpha - R_t^d d_t \right\}^+ dF(\varepsilon). \tag{3}$$

Equation (3) implies that $\underline{\varepsilon}_{t+1}$, the highest value of $\varepsilon$ at which banks default on their depositors, is implicitly defined as a function of $A_{t+1}$:

$$R_t^d d_t = A_{t+1}\underline{\varepsilon}_{t+1} k_t^\alpha. \tag{4}$$

### 3.1.3 Government

The government taxes agents' wealth $x_{t+1}^i$ at a rate $\tau_{t+1}$ in order to ensure that depositors at failed banks at the beginning of $t + 1$ are made whole. The government seizes output at failed banks (who return zero to their equity-holders) to partially defray the expenses of paying back depositors, but its recovery efforts are subject to a deadweight loss that is quadratic in the amount of output to be collected.

The total amount of tax to be collected is

$$T_{t+1} = \int \left\{ \underbrace{R_t^d d_t}_{\text{owed to depositors}} - \underbrace{A_{t+1}\varepsilon k_t^\alpha}_{\text{collected from banks}} \right\}^+ dF(\varepsilon) + \frac{\lambda}{2} \underbrace{\left[ \int_{-\infty}^{\underline{\varepsilon}_{t+1}} A_{t+1}\varepsilon k_t^\alpha dF(\varepsilon) \right]^2}_{\text{deadweight loss}}$$

where the parameter $\lambda$ in the second term indexes the extent of deadweight losses in the economy. Then the tax rate on wealth $\tau_{t+1}$ satisfies

$$\tau_{t+1} = \frac{T_{t+1}}{\int \left[ \left(z_{t+1}^i - w_t\right) l_t^i + R_t^d d_t^i + R_{t+1}^E n_t^i \right] di} \tag{5}$$

where the denominator is aggregate pre-tax wealth across all agents $i$. $\tau_{t+1}$ depends on the realization of the aggregate shock $A_{t+1}$, which affects both the realized return on equity and the fraction of banks that default in equilibrium.

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### 3.1.4 Laborers

Laborers live for a single period and are hand-to-mouth; they choose consumption $c_t$ and labor supply $l_t$ to solve

$$\max_{c_t, l_t} \quad c_t - \nu_1 \frac{l_t^{1+\frac{1}{\nu_2}}}{1+\frac{1}{\nu_2}} \tag{6}$$

subject to the budget constraint

$$c_t = w_t l_t.$$

## 3.2 Equilibrium Definition

Given exogenous stochastic processes for the aggregate productivity $A_t$ and $z_t^i$, equilibrium is a collection of firm policies, bank policies, and government taxes such that

1. Agents' choices for labor demand $l_t^i$, deposits $d_t^i$, equity investment $n_t^i$, and consumption $c_t^i$ maximize their utility (1);

2. Banks' choices for capital $k_t$ and deposits $d_t$ solve their problem (2),

3. Bank profits are returned to agents holding bank equity through the return on equity given in equation (3);

4. The government taxes agents in proportion to their wealth and uses the proceeds to pay depositors at failed banks according to equation (5).

5. Laborers' labor supply is consistent with maximizing their utility (6).

6. The wage $w_t$ and the return on deposits $R_t^d$ clear the labor and deposit markets, respectively.

# 4  Results

We begin our analysis of the model by showing that entrepreneurs' choices can be easily aggregated despite the heterogeneity in their wealth (Section 4.1). We then present our main result about the effects of capital requirements on the good risk-taking channel of entrepreneurs (Section 4.3); to clarify the exposition, we set the deadweight loss of default to zero (i.e., $\lambda = 0$) so that we can ignore the more standard bad risk-taking channel of banks. We then consider a version of the model where both the good and bad risk-taking channels are at work and capital requirements trade off these two effects (Section 5).

## 4.1  Entrepreneurs' and laborer' choices, and aggregation

The following proposition greatly simplifies the analysis by allowing us to aggregate easily across entrepreneurs.

**Proposition 1.** *Agent i's optimal choices are given by*

$$c_t^i = (1 - \beta)\, x_t^i$$

$$l_t^i = \phi_t \beta x_t^i$$

$$n_t^i = y_t \beta x_t^i$$

$$d_t^i = (1 - y_t)\, \beta x_t^i$$

*where $\phi_t$ and $y_t$ are independent of $x_t^i$ and satisfy the following first-order conditions:*

$$0 = E_t \left\{ \frac{z_{t+1}^i - w_t}{\Delta_{t+1}^i} \right\} \tag{7}$$

$$0 = E_t \left\{ \frac{R_{t+1}^E - R_t^d}{\Delta_{t+1}^i} \right\} \tag{8}$$

*where*

$$\Delta_{t+1}^i \equiv \left( z_{t+1}^i - w_t \right) \phi_t + y_t R_{t+1}^E + (1 - y_t) R_t^d. \tag{9}$$

*Proof.* See Appendix A. □

A key result of Proposition 1 is that the tax rate $\tau_{t+1}$ does not enter agents' first-order conditions. In addition, the total savings of entrepreneurs, $\int \left[ d_t^i + n_t^i \right] di = d_t + n_t$ are a constant fraction $\beta$ of aggregate wealth $X_t \equiv \int x_t^i di$, so that total investment by banks $k_t = d_t + n_t$ is also a constant fraction of entrepreneurial wealth in equilibrium. Thus, for a given $X_t$, the risk-shifting activity of banks due to deposit insurance affects their default probability and the deposit and equity returns, but not the size of their balance sheet.

In the remainder of the paper, $y_t$ is the fraction of agents' savings $\beta x_t^i$ devoted to bank equity, and $\phi_t$ is their labor demand as a proportion to their savings.

Because agents' choices are all proportional to their individual wealth $x_t^i$, aggregates don't depend on the distribution of $x_t^i$ across agents. Thus aggregate labor demand is given by

$$
\begin{aligned}
L_t &= \int l_t^i di \\
&= \int \phi_t \beta x_t^i di \\
&= \phi_t \beta X_t,
\end{aligned}
\tag{10}
$$

and aggregate wealth next period is

$$X_{t+1} = (1 - \tau_{t+1}) \left[ \left( \bar{z}_{t+1} - w_t \right) \phi_t + y_t R_{t+1}^E + (1 - y_t) R_t^d \right] \beta X_t \tag{11}$$

where $\bar{z}_{t+1}$ is the average idiosyncratic shock across agents.

The first-order condition to the laborer's problem (6) implies that the labor supply curve

is given by

$$w_t = \nu_1 \left(L_t\right)^{\frac{1}{\nu_2}}. \tag{12}$$

## 4.2 Capital requirements and bad risk taking

It is straightforward to show that the first-order condition to the bank's problem (2) is given by

$$\alpha k_t^{\alpha-1} E_t \left\{ A_{t+1}\varepsilon_{t+1} \left| \varepsilon_{t+1} > \underline{\varepsilon}_{t+1} \right.\right\} = R_t^d + \xi, \tag{13}$$

where $\xi$ is the Lagrange multiplier on the capital requirement constraint.

Equation (13) illustrates the bad risk-taking aspect of capital requirements. If the capital requirement constraint does not bind, and banks have a positive probability of default, then the expectation term on the left-hand-side of equation (13) must be strictly greater than the expected value of $A_{t+1}$. Even though $\varepsilon$ is an idiosyncratic shock, due to limited liability bankers only consider future states in which they are solvent when they decide how much to borrow and invest.

Suppose $R_t^d$ were a fixed constant, as it would be in a model with risk-neutral investors such as the illustrative model of Davydiuk (2017). In order for equation (13) to be satisfied, bankers need to increase their investment in $k_t$ relative to what a social planner would choose. Moreover, as Davydiuk (2017) makes clear, the degree of overinvestment depends in part on the state of the business cycle, through the distribution of future aggregate productivity $A_{t+1}$. However, capital requirements can overcome this inefficiency by making the constraint bind, increasing the value of $\xi$ until banks choose the desired level of investment $k_t$.

In our model, Proposition 1 implies that the level of bank investment $k_t$ is pinned down by agents' savings behavior, which is a constant fraction of aggregate wealth $X_t$. In this case, when the capital requirement does not bind, equation (13) is satisfied by increasing the

value of $R_t^d$ in equilibrium. Banks would like to over-invest due to limited liability and their positive default probability, but in equilibrium their attempts to do so only raise the return on deposits. Increasing capital requirements in our economy will reduce the deposit return $R_t^d$, but will not affect total savings or bank investment.

However, the fact that capital requirements in our model only affect $R_t^d$ in equilibrium does not mean that they have no real effects. In our model, changing the return on deposits does not increase savings in bank deposits or equity; these are a constant fraction of wealth, according to Proposition 1. However, changing the return on deposits *does* affect the labor demand of agents in equilibrium, through equations (7) and (9). In fact, as we show in the next section, the "distorted" return on deposits in equation (13) is a "good" distortion, in that it raises total output by inducing agents to take more idiosyncratic risks, and higher capital requirements reduce this "good" risk taking.

## 4.3   Capital requirements and good risk taking

In this section, we provide our main results about the effects of capital requirements on good risk taking by entrepreneurs. To clarify the exposition, we focus on the simple case in which aggregate productivity is constant (i.e., $A_t = A$ for all $t$) and there are no deadweight losses associated with bank default (i.e., $\lambda = 0$). Because of the assumption of constant $A_t$, we can focus on a steady state in which all aggregate variable are constant as well.

We first present the effects of capital requirements on aggregate wealth $X_t$ and agents' consumption in Proposition 2, and then we turn to welfare in Proposition 3. We distinguish between two cases: constant $z_{t+1}^i$ and stochastic $z_{t+1}^i$.

**Proposition 2.** *Suppose $\lambda = 0$ and $A_{t+1} = A$ is not random. Then if $z_{t+1}^i$ is a known constant for all $t$ and all $i$, changing the capital requirement $\zeta$ has no effect on aggregate wealth $X_t$ and agents' consumption. On the other hand, if $z_{t+1}^i$ is random, increasing $\zeta$ when the capital requirement constraint binds reduces aggregate wealth $X_t$ and agents' consumption in steady-state.*

*Proof.* See Appendix A. □

Proposition 2 characterizes the real effects of capital requirements, in the way they affect agents' idiosyncratic risk-taking. Capital requirements force banks to hold more equity, which reduces their default probability in equilibrium. But because $\lambda = 0$, the level of bank default is irrelevant: a reduction in bank defaults reduces the amount that the government must tax agents to pay for deposit insurance, but this reduction in the tax is exactly offset by a reduction in the return to deposits (and to bank equity).

Thus if agents face no idiosyncratic risk, capital requirements have no real effects. However, when agents face idiosyncratic productivity shocks, the reduction in the return on deposits has the effect of increasing the risk agents face: the risks in their idiosyncratic labor income have not changed, but the share of their income from safe bank deposits has gone down. Because agents are risk-averse, they reduce their labor demand in response. This lowers aggregate output and steady-state wealth, making agents worse off.

Figure 1 illustrates the basic mechanism in the model. The top panel plots the equilibrium in the deposit market, where the red line and blue lines plot equations (3) and (8), respectively, in the $\left(R^d, R^E\right)$ plane. Because there is no aggregate risk, equation (8) implies that $R^E = R^d$ and therefore the red line is a 45-degree line in the $\left(R^E, R^d\right)$ plane. On the other hand, by equation (3) it must be that $R^E$ is decreasing in $R^d$ from the bank's perspective, which gives the blue curves in the top panel of Figure 1.

Increasing the capital requirement $\zeta$ does not affect the link between $R^E$ and $R^d$ implied by entrepreneurs' first-order condition (8), but it changes the return on equity payed by banks. Thus, the red line in the top panel of Figure 1 is unchanged, whereas the blue line shifts from the solid to the dotted one. To see this, rewrite equation (3) as

$$
\begin{aligned}
R^E &= \frac{1}{n} \int \left\{ A\varepsilon k^\alpha - R^d d \right\}^+ dF\left(\varepsilon\right) \\
&= \frac{1}{\zeta} \int_{\underline{\varepsilon}}^\infty \left[ A\varepsilon k^{\alpha-1} - (1-\zeta) R^d \right] dF\left(\varepsilon\right)
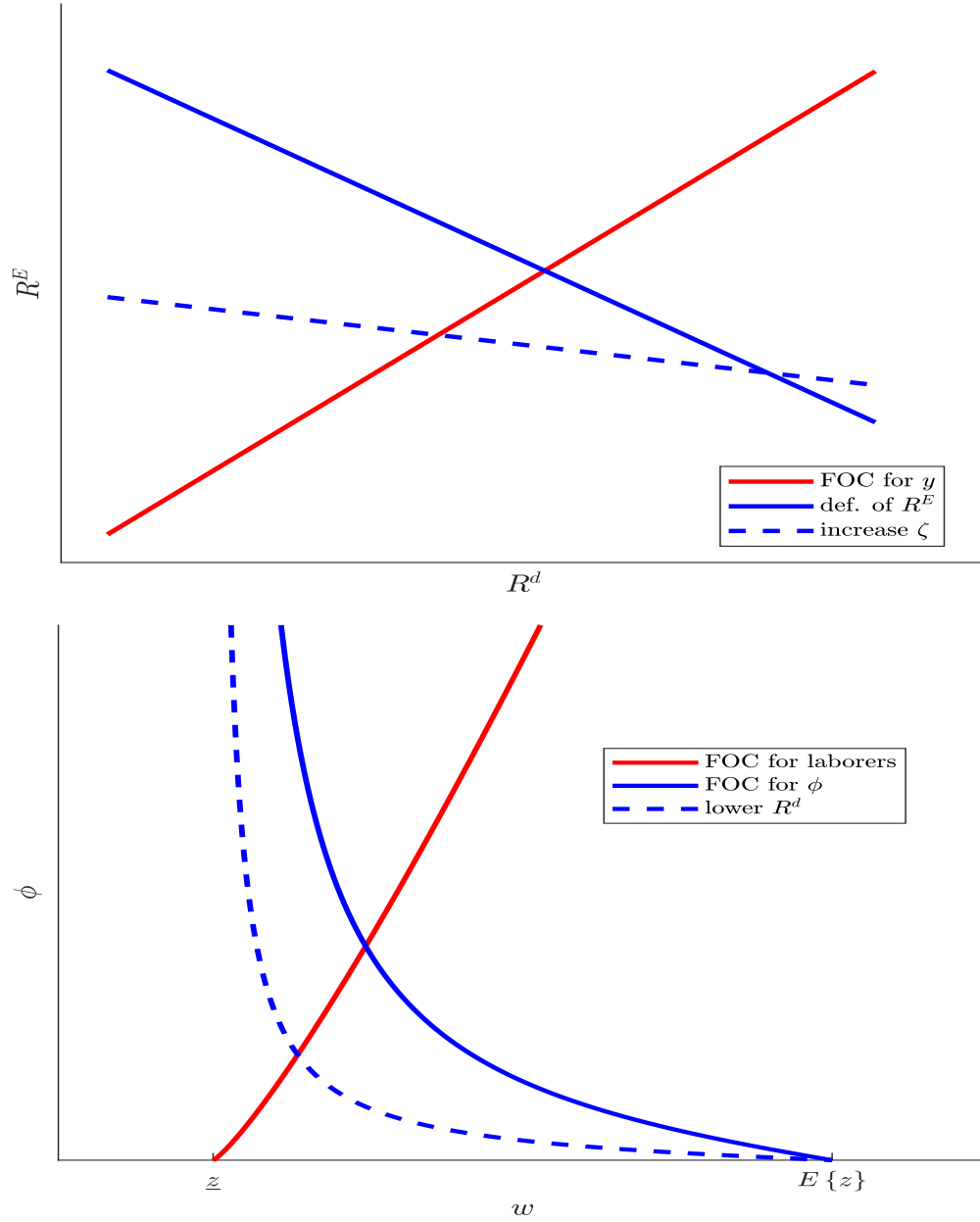\end{aligned}
$$

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**Figure 1.** Equilibrium in the Asset and Labor Markets

The top panel plots the equilibrium in the asset market. The $x$- and $y$-axes are the returns on deposits $R^d$ and equity $R^E$, respectively. The red line plots equation (8) and the blue lines plot equation (3) for two binding values of $\zeta$. The dotted blue line represents a higher, binding value of $\zeta$ than the solid blue line. The bottom panel plots the equilibrium in the labor market. The red line plots the labor-supply curve (12), while the blue lines plot the labor-demand curve (7) for a fixed value of $X$. The solid blue line plots labor demand for one value of $R^d$, while the dotted line plots labor demand for a lower value of $R^d$.

where the second line plugs in the constraints $n/k = \zeta$ and $d = k - n = (1 - \zeta)\,k$. Taking the derivative of $R^E$ with respect to $\zeta$ yields

$$\frac{\partial R^E}{\partial \zeta} = -\frac{1}{\zeta^2} \int_{\underline{\varepsilon}}^{\infty} \left[ A\varepsilon k^{\alpha-1} - R^d \right] dF(\varepsilon) \tag{14}$$

where there is no $\partial\underline{\varepsilon}/\partial\zeta$ term because the integrand in equation (3) is zero (by definition) at $\underline{\varepsilon}$. The sign of the derivative in equation (14) can be either positive or negative, as shown in the top panel of Figure 1, but it must be negative for values of $R^d$ close to $R^E$. This is the content of Proposition 2. Intuitively, raising the capital requirement increases the number of equity holders who must be paid out of bank profits, lowering $R^E$, but it also reduces the number of depositors who must be paid $R^d$. If $R^d$ were very high, this second effect could dominate and increasing $\zeta$ would increase $R^E$; but this cannot occur in equilibrium where $R^E = R^d$.

The bottom panel of Figure 1 plots the equilibrium in the labor market. By equation (10), labor demand depends both on aggregate agent wealth $X_t$ as well as their proportional labor choice $\phi$, which solves equation (7) when $R^E = R^d$ by equation (8). It is straightforward to show that $\phi = \infty$ if $w \le \underline{z}$, the lowest possible value of $z$, and that $\phi = 0$ when $w = E\{z\}$, so that in equilibrium $w$ must lie between these two values. In addition, in equilibrium $\phi$ scales with $R^d$, so that a decrease in $R^d$ (such as the one plotted in the top panel of Figure 1) lowers agents' proportional labor demand $\phi$ in equilibrium. In turn, a lower value of $\phi$, combined with the fact that $\lambda = 0$ and thus no output is saved from the reduction in bank defaults, implies a reduced steady-state value of $X_t$ and thus lower labor demand and agents' consumption.

Next, we show that tightening the capital requirement constraint reduces welfare. Before presenting the results, we explain how we measure welfare, given that there are two dimensions of heterogeneity: different wealth among entrepreneurs, and two classes of agents (i.e., entrepreneurs and laborers).

Entrepreneurs are heterogenous because of the effects of productivity shocks on their wealth accumulation. We define the welfare of entrepreneurs as their value function $V(x)$ evaluated at the average wealth in the economy in steady state, $x = X$. Formally, this measure of welfare would arise in an economy in which all agents have the same wealth at $t = 0$, $x_0^i = X_0$, so that $\int V(x_0^i) di = V(X_0)$, and $X_0$ is initialized at the steady-state level. Thus, this measure of welfare does not account for the effects of policy on the distribution of wealth. A more general measure of welfare can be derived by extending the model to obtain a well-defined stationary distribution of entrepreneurs' wealth, and by integrating $V(x_0^i)$ with respect to such a stationary distribution. If tightening capital requirements increases the dispersion of wealth among entrepreneurs, our conclusions would be reinforced.

The fact that our model includes two classes of agents – entrepreneurs and laborers – does not affect the welfare results of this section. We show that the welfare of both entrepreneurs and laborers decreases when capital requirements are binding and they are tightened.

The next proposition formalizes the effects of capital requirements on welfare through the good-risk taking channel, when we shut down the bad risk-taking channel by setting $\lambda = 0$.

**Proposition 3.** *Suppose $\lambda = 0$ and $A_{t+1} = A$ is not random. Then if $z_{t+1}^i$ is a known constant for all $t$ and all $i$, changing the capital requirement $\zeta$ has no effect on welfare of agents and laborers. On the other hand, if $z_{t+1}^i$ is random, increasing $\zeta$ when the capital requirement constraint binds reduces the welfare of both agents and laborers.*

*Proof.* See Appendix A. □

The results of Proposition 3 follows from those of Proposition 2. With no idiosyncratic risk, Proposition 2 shows that capital requirements have no real effects, and thus welfare must be unchanged. If instead agents are subject to idiosyncratic risk, the reduction in steady-state value of $X_t$ reduces welfare. Given that our measure of welfare is equivalent to $V(X_0)$ with $X_0$ initialized at the steady-state level of $X_t$, welfare moves one-for-one with $X_t$.

# 5 Numerical Examples

In this section we explore two calibrated versions of the model, in order to illustrate how capital requirements can balance the good risk-taking investments of entrepreneurs against the bad risk-taking decisions of banks. In Section 5.1 we assume that the idiosyncratic bank shock $\varepsilon$ is log-normally distributed and that aggregate TFP $A$ is constant. In this case increasing capital requirements quickly reduces bank default probability, so that a relatively low capital requirement quickly eliminates both costs and benefits of capital requirements. Nevertheless we show that there is an optimal capital requirement for this model.

In section 5.2 we add an aggregate shock representing a financial crisis: in "normal" times $\varepsilon$ is a constant, but in 'crisis" times $\varepsilon$ takes one of two values for each bank. In this case, only very high capital requirements can eliminate bank risk; but capital requirements nevertheless reduces the good risk-taking of entrepreneurs and the deadweight loss from banks' bad risk-taking. We calibrate this "large-shock" model the Savings & Loan crisis of the late 1980s and early 1990s and show that the implied optimal capital requirement is much larger than for the continuous-shock model of Section 5.1.

## 5.1 Continuous-Shock Model

In this section we assume that the bank's idiosyncratic shock $\varepsilon$ is lognormally distributed. We lightly abuse notation by replacing $\varepsilon$ everywhere with $\exp\left\{\sigma\varepsilon - \frac{1}{2}\sigma^2\right\}$, where now $\varepsilon$ is a standard normal random variable.

We solve the model numerically, assuming that $A$ is constant, $\varepsilon$ is a standard normal random variable, and that $z_{t+1}^i$ is i.i.d. and can take two values, $z^H$ with probability $p$ and $z^L < z^H$ with probability $1 - p$. In this case, because there is no aggregate risk, there is an aggregate steady-state in which all aggregate variables are constant, and only agent-level wealth and productivity fluctuate. Thus in what follows we drop $t$ subscripts where convenient. See Appendix B for details on the computation of the model solution.

With lognormal shocks, it can be shown (see equation 28 in Appendix B) that the agent's total return on savings is given by

$$\underbrace{yR^E + (1-y)R^d}_{\text{return on savings}} = Ak_t^{\alpha-1}\underbrace{\left[1 - \Phi\{\underline{\varepsilon} - \sigma\} + e^{\sigma\varepsilon}\Phi\{\underline{\varepsilon}\}\right]}_{\text{deposit insurance subsidy}} \qquad (15)$$

where $\underline{\varepsilon}$ is the idiosyncratic bank shock below which banks default. The top panel of Figure 2 plots the terms that depend on $\underline{\varepsilon}$ as a function of the default probability, $\Phi\{\underline{\varepsilon}\}$ where $\Phi\{\cdot\}$ is the standard normal CDF, for various values of $\sigma$.

As can be seen in the top panel of Figure 2, the deposit insurance subsidy in equation (15) is an increasing function of both the default probability and the amount of idiosyncratic risk $\sigma$. Thus, increasing capital requirements (which, when binding, always reduce the default probability of banks in this model) reduces the subsidy to savings that agents enjoy. Even though this subsidy is paid for out of taxes on agents, the subsidy appears in agents' first-order conditions (7) and (8), while the tax rate $\tau$ does not. This is the content of Proposition 2; the top panel Figure 2 illustrates the size of this effect, as a multiple of marginal bank output, for a range of values of the default probability and $\sigma$.

To further illustrate the workings of the model, we calibrate the model to match some numerical targets and plot how steady-state wealth varies with capital requirements. Increasing aggregate wealth is commensurate with increasing social welfare in this model because higher aggregate wealth $X_t$ leads to higher labor demand and thus higher wages $w_t$; thus both laborers and agents are strictly better off.

Panels A and B of Table 1 report the parameter values we used for this exercise, as well as the numerical targets we chose. We set the Frisch elasticity $\nu_2$ to be very high to keep the wage from responding too much "against" the agents; fr low values of $\nu_2$, most adjustment to capital requirements occurs in wages rather than in the quantity of labor, and the effects on output are small.

Even though this is just a numerical example, we want to comment on our choice of
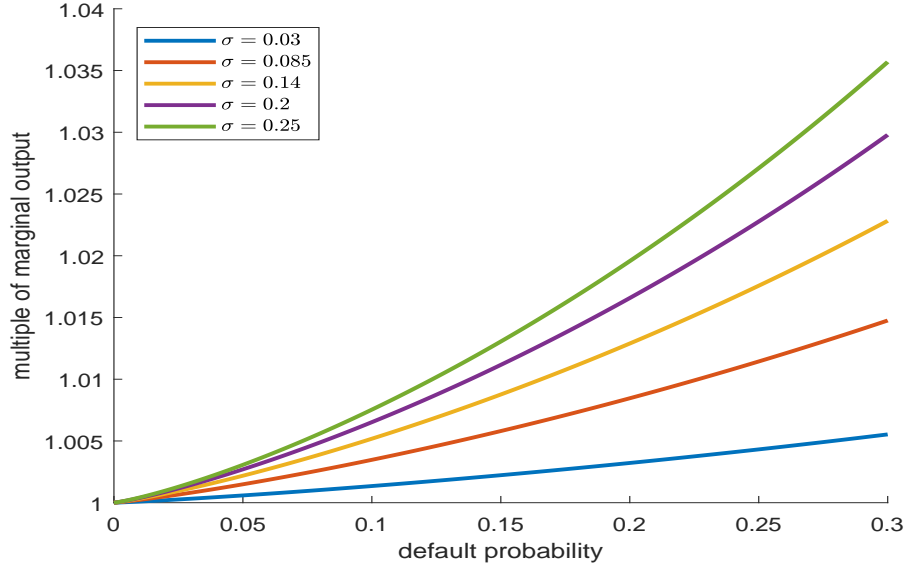
(a) Deposit Insurance Subsidy
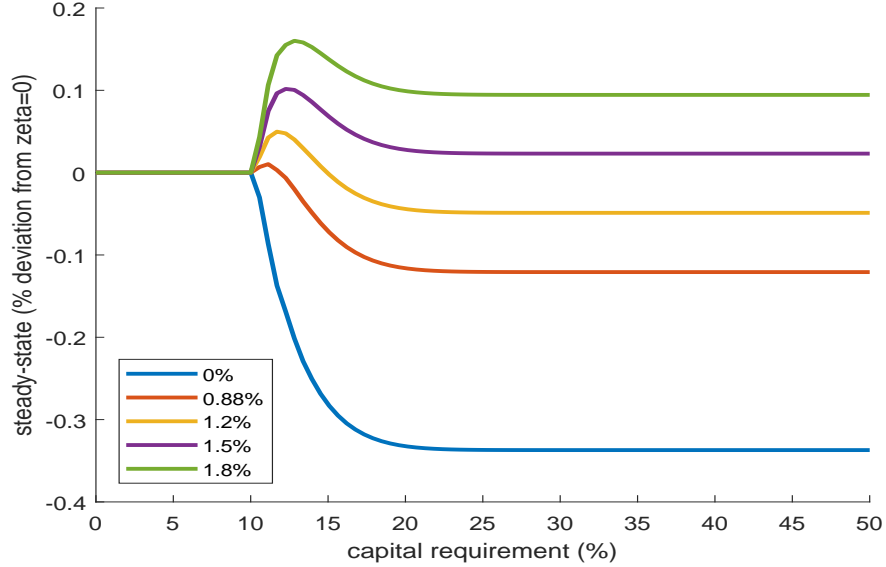


(b) Steady-State Wealth

**Figure 2.** Continuous-Shock Model

The top panel plots the deposit insurance subsidy as a function of the default probability $\Phi\{\underline{\varepsilon}\}$ from equation (15) for various values of $\sigma$ ranging from 0.03 to 0.25. The bottom panel plots steady-state aggregate wealth $X_t$, in percent deviations from the no capital requirement equilibrium ($\zeta = 0$), as a function of the capital requirement $\zeta$ for values of the deadweight-loss parameter $\lambda$ ranging from 0 to 15. The percentages reported in the bottom-left corner of the lower panel are the deadweight loss as a percentage of GDP. The parameters used for these calculations are reported in Table 1, panels A and B.

| Panel A: Common Parameters | |
| --- | --- |
| Parameters | Value |
| $\beta$ | 0.95 |
| $\nu_2$ | 100 |
| $z^H$ | $A$ |
| $z^L$ | 0 |
| $\Pr\left\{z = z^H\right\}$ | 0.70 |

| Panel B: Continuous Bank Shocks | | | |
| --- | --- | --- | --- |
| Parameters | Value | Target | Target Value |
| $\alpha$ | 0.98 | | |
| $A$ | 0.5651 | Steady-State $X$ | 1.053 |
| $\delta$ | 0.5370 | Unconstrained Default Probability $\Phi\left\{\underline{\varepsilon}_{t+1}\right\}$ | 10% |
| $\sigma$ | 0.077 | Unconstrained Capital Ratio $y$ | 10% |
| $\nu_1$ | 0.359 | Deposit Premium $\frac{1}{\beta} - R^d$ | 2% |

| Panel C: Large Bank Shocks | | | |
| --- | --- | --- | --- |
| Parameters | Value | Target | Target Value |
| $p_c$ | 1% | Financial Crisis Probability | 1% |
| $s$ | 8.9% | Share of Bank Failures in S&L Crisis | 8.9% |
| $A$ | 1.032 | Steady-State $X$ | 1.053 |
| $\alpha$ | 0.99989 | Unconstrained Capital Ratio $y$ | 10% |
| $\nu_1$ | 0.6612 | Deposit Premium $\frac{1}{\beta} - R^d$ | 2% |
| $\gamma$ | 0.66 | Tax/GDP during Financial Crisis | 1.7% |

**Table 1.** Numerical Example Parameter Values
The table reports the parameter values used in Figures 2 and 3. Panel A reports parameters common across both the continuous-shock and the large-shock models. These parameters are not set to match any targets. Panel B reports parameters for the continuous-shock model along, with the calibration targets for each parameter (apart from $\alpha$, which is set without a target). Panel C reports the parameters used for the large-shock model along with the corresponding calibration targets.

$\alpha$. Since the production function of banks does not include labor, the parameter $\alpha$ can be interpreted as the degree of decreasing return to scale for banks' assets. With this in mind, our choice is in line with the value of 0.85 used by Midrigan and Xu (2014). In addition, plugging equation (30) into (34) and rearranging reveals that the default probability of banks must be greater than $1 - \alpha$. Thus to keep the default probability at a low level, we keep $\alpha$ relatively high.

In order to ease the calibration, for this model we allow bank assets to not depreciate fully; that is, we replace the banker's objective in equation (2) with

$$\max_{k_t, d_t} \ E_t \int \left\{ \left[ A_{t+1} k_t^\alpha + (1 - \delta) k_t \right] \varepsilon - R_t^d d_t \right\}^+ dF(\varepsilon), \tag{16}$$

where $\delta$ is the rate at which bank capital depreciates. Equation (16) assumes that the idiosyncratic bank shock hits both bank output $A_{t+1} k_t^\alpha$ as well as underpreciated capital $(1 - \delta) k_t$; in that sense, $\varepsilon$ is more like a "capital-quality" shock than a productivity shock.

Panel B of Table 1 reports the parameters that were chosen to match numerical targets for the continuous-shock model. We set $\nu_1$ to match a deposit premium $1/\beta - R^d$ of 2%. $\nu_1$ primarily affects the deposit premium in equilibrium by reducing the level of wages; holding agent productivity fixed, lower wages mean that the idiosyncratic project income makes up a larger share of agent income, which means that the return on deposits (and equity) must be lower in order for next period's aggregate wealth to equal this period's aggregate wealth (steady-state).

We choose $\sigma$ in order to match a bank default probability of 10%. Given all other parameters and their targets, we vary $A$ and $\delta$ until the unconstrained bank first-order condition (13) leads to a 10% capital ratio, and steady-state aggregate wealth $X_t$ is $1/\beta$. We choose this particular value of steady-state unconstrained aggregate wealth for numerical convenience.

The bottom panel of Figure 2 plots steady-state aggregate wealth $X_t$ as a function of

capital requirements for various values of the deadweight-loss parameter $\lambda$ ranging from 0 to 15. Capital requirements less than 10% do not bind and thus have no effect on aggregate wealth. As shown in Proposition 2, when $\lambda = 0$ capital requirements only deter good risk-taking and reduce aggregate wealth.

When $\lambda > 0$, bank defaults entail a deadweight loss and capital requirements can be valuable in reducing this deadweight loss. Then the optimal capital requirement balances a reduced deadweight loss with the costs associated with lower labor demand from agents.We show this effect in the bottom panel of Figure 2, where the box in the bottom left corner report the deadweight loss as a percentage of GDP when the capital requirement is zero. As $\lambda$ goes up, potential deadweight loss increases, and capital requirements can avert this deadweight loss. Higher values of $\lambda$ imply a higher optimal capital requirement, although the optimum is never very far from the unconstrained value of 10%.

## 5.2 Large-Shock Model

In this section we replace the lognormal shocks of the model in section 5.1 with "large" shocks; that is, now we assume that $\varepsilon$ takes one of two values:

$$\varepsilon_{t+1} \sim \begin{cases} \gamma & \text{probability } p_{t+1} \\ \frac{1-\gamma p_{t+1}}{1-p_{t+1}} & \text{probability } 1 - p_{t+1} \end{cases},$$

where $p_{t+1}$ is itself random, taking one of two possible values:

$$p_{t+1} \sim \begin{cases} 0 & \text{probability } 1 - p_c \\ s & \text{probability } p_c \end{cases}.$$

In this model, with probability $1 - p_c$ we are in "normal" times and no banks will default. However, with a very low probability $p_c$, banks are subjective to an idiosyncratic shock, and a fraction $s$ of them will only produce a fraction $\gamma$ of their output. The remaining banks will

produce somewhat more, in order for the shock to remain idiosyncratic (that is, $E\{\varepsilon\} = 1$).

We interpret this model set-up as representing infrequent but large banking crises: agents do not know how many (or if any) banks will fail next period, but there is always a possibility of disaster. In addition, because we have assumed that $p_{t+1}$ is iid over time, aggregate wealth $X_t$ will be the only aggregate state variable in this economy.

Notice also that in this model, capital requirements that are far from $1 - \gamma$ will not affect the probability of bank default, which remains at $p_{t+1}$. Only if banks have enough capital so that

$$y > 1 - \gamma \frac{Ak_t^{\alpha-1}}{R_t^d} \tag{17}$$

will banks survive receiving the low value of $\varepsilon$. If capital requirements are high enough that equation (17) is satisfied, then the default probability of banks will be zero; otherwise, it will be $p_{t+1}$.

As in the model of section 5.1, we solve this model numerically, assuming constant $A$, using the parameters reported in Panels A and C of Table 1. Our calibration targets in this case are chosen to match some features of the Savings & Loan Crisis (S&L) of the late 1980s and early 1990s, in which over a third of savings & loans covered by the Federal Savings & Loan Insurance Corporation (FSLIC) failed.

We set $s = 8.9\%$, in order to match the fraction of banks that defaulted during the S&L crisis. We arrive at this figure by including the 1,043 of 3,324 S&Ls that failed during the crisis (Curry and Shibut 2000) and a default probability of 2.6% that applied to the roughly 12,000 FDIC-insured banks at the time.[5] We set $p_c = 1\%$. We set $\alpha$ in order to keep the unconstrained bank capital ratio at 10%, as in the model of section 5.1. Finally, we set the

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[5]The FDIC figures come from the FDIC Historical Statistics on Banking, available at https://www5.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10&Header=1. The calculation is then

$$s = \frac{1,043 + 2.6\% \times 12,000}{3,324 + 12,000} \approx 8.9\%$$

bad-shock output value $\gamma$ in order to match the total cost of the S&L crisis that was borne by taxpayers, as a share of GDP. According to General Accounting Office (1996), this was roughly 1.7% of GDP.[6]

It is straightforward to show in this model that the total return on agents' savings is given by

$$y_t R_{t+1}^E + (1 - y_t) R_t^d \tag{18}$$
$$= A_{t+1} k_t^{\alpha-1} + \underbrace{p_{t+1} \left[ (1 - y_t) R_t^d - \gamma A_{t+1} k_t^{\alpha-1} \right]}_{\text{deposit insurance subsidy}},$$

where $y_t$ is the share of their wealth that agents invest in bank equity. As in equation (15), when the bank default probability is zero the total return on savings is the equal to the total return on bank assets, irrespective of how much equity or debt agents hold. This is true even when $A_{t+1}$ is risky; giving agents access to a riskless asset through deposit insurance has no effect in equilibrium because they hold both the debt and the equity of banks. On the other hand, when $p_{t+1}$ is positive, deposit insurance is a subsidy to agents that increases the return on their savings. Although this subsidy will be paid for with taxes on their wealth, it affects real output by increasing agents' "good" risk taking in their idiosyncratic project.
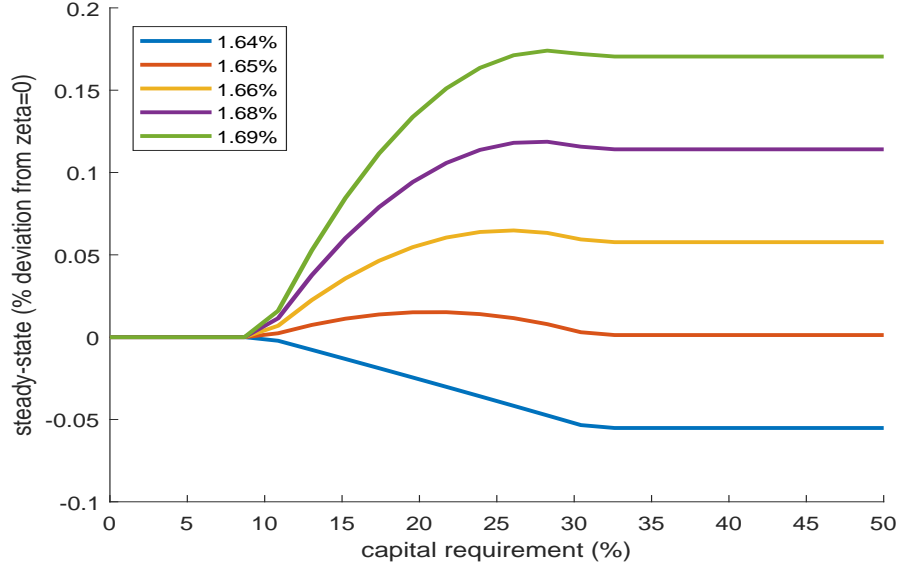
Figure 3 plots steady-state aggregate wealth $X_t$ for this model, in percentage deviations from the no capital requirements case, as a function of capital requirements and for various values of $\lambda$ ranging from 0 to 3. When $\lambda = 0$, there are no deadweight losses from bank default and thus no benefit to higher capital requirements; capital requirements only reduce the savings subsidy, and with it, agents' investment in labor. Output and wealth fall as capital requirements increase, until the capital requirement is high enough (around $\zeta = 34\%$) that banks that receive the low value of $\varepsilon$ have enough capital that they do not default.

At higher values of $\lambda$, capital requirements above the unconstrained capital ratio of 10% are helpful in reducing deadweight losses from bank default. The percentages in the lower

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[6]$132 billion; US nominal GDP in 1995 was roughly $7.66 trillion.
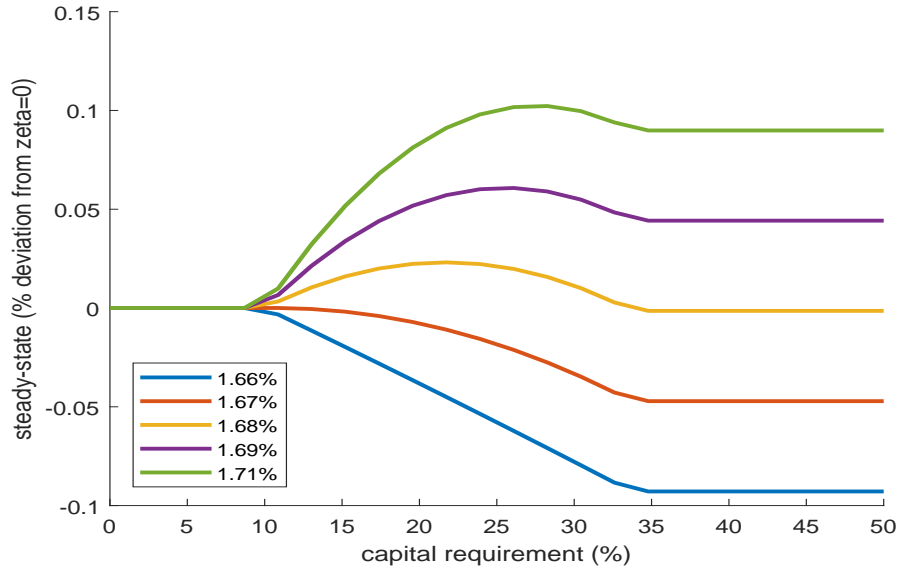
(a) High Deposit Premium (2%)



(b) Low Deposit Premium (1%)

**Figure 3.** Large-Shock Model

The top panel reports the steady-state aggregate wealth $X_t$, in percent deviations from the no capital requirement equilibrium ($\zeta = 0$), as a function of the capital requirement $\zeta$ for values of the deadweight-loss parameter $\lambda$ ranging from 0 to 3. The percentages reported in the bottom-left corner of the lower panel are the total tax (and not just the deadweight loss) as a percentage of GDP. The parameters used for these calculations are reported in Table 1, panels A and C. The bottom panels repeats the same calculations but changes the values of $\nu_1$ to 0.7119, $\alpha$ to 0.99988, and $\gamma$ to 0.6978, which allows the model to match a deposit premium of 1% but still match the other targets given in Table 1, panel C.

left corner of the top panel of Figure 3 report the total tax, as a percentage of output, borne by agents in the crisis state. Higher values of $\lambda$ increase this tax, but for all values considered in Figure 3 the tax is still very close to the 1.7% of GDP calibration value.[7] As $\lambda$ increases, the optimnal capital requirement rises, to roughly 27% for $\lambda = 3$ (green line).

Notice that in this model, 99% of the time there are no bank failures and the government does not tax agents at all. Aggregate wealth $X_t$ changes every period, but because the crisis shock is iid, agents' decisions and all other endogenous aggregate variables are constant over time. Although a banking crisis where a fraction $s$ of banks onlty return $\gamma$ happens with only 1% probability, agents fear that state and choose less labor because of it. Deposit insurance acts to insure them against their own idiosyncratic risk: it raises the total return on their savings (equation 18), but because the tax is proportional to their wealth, the cost of this subsidy is borne more by agents whose projects are successful and earn the high value of $z'$. This indirect insurance on their idiosyncratic risk induces agents to take on more "good" risks in equilibrium, and increase aggregate output even though the taxes and insurance only materialize 1% of the time.

The bottom panel of Figure 3 repeats the same exercise, but re-calibrates the model to match a deposit premium $1/\beta - R^d$ of 1%, rather than the baseline of 2% in panel C of Table 1. We match the deposit premium by varying the level of wages, though the intercept of the labor-supply curve $\nu_1$. Increasing average wages reduces the value of agents' indiosyncratic projects, which in steady-state brings the deposit return closer to the rate of time preference $1/\beta$. To continue to match the other calibration targets in panel C of Table 1, we also change $\alpha$ (the degree of decreasing returns to scale in banks' assets) and $\gamma$ (output at bad banks during the crisis).

Reducing the value of the idiosyncratic labor projects reduces the cost of capital requirements, but not the benefits: in the bottom panel of Figure 3, there is less output lost when $\lambda = 0$, and more output gained when $\lambda > 0$, at every value of (binding) capital require-

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[7]This is why we set $\gamma$ to match the total tax borne by agents, and not $\lambda$. Finding an appropriate data moment to which we can calibrate $\lambda$ is a challenge for this model.

ments. The optimal capital requirement for each level of $\lambda > 0$ increases, and in fact the lost output from setting capital requirements higher than the optimum os almost zero. For this calibration, the cost of setting capital requirements "too high" is virtually zero. Thus the value of the deposit premium to which we calibrate the model is crucial for understanding the costs of capital requirements in this model.

# 6  Conclusion

We propose a new channel through which capital requirements affect the aggregate economy by changing the volatility of the savings of entrepreneurs. Higher capital requirements reduce the returns on banks liabilities, which increase the volatility of entrepreneurs' savings and induce them to reduce their labor demand in equilibrium, lowering output and consumption. Optimal capital requirements balance the reduction in output from this "good" risk-taking channel with a reduced deadweight loss from bank default, a standard "bad" risk-taking channel.

Our analysis thus far has been qualitative, but we plan to extend the paper to assess quantitatively the magnitude of the good risk-taking channel. We have purposefully constructed the model to maintain tractability for any stochastic process for the productivities $A_{t+1}$ and $z_{t+1}^i$, including making $A_{t+1}$ persistent. This is important because, as shown for example by Davydiuk (2017), optimal capital requirements may be time-varying either because first-best investment varies with the state of the business cycle, or because the extent of excessive bank risk-taking does (or both).

# A Proofs

## Proof of Proposition 1

To ease notation we remove all $t$ subscripts, and put a prime $'$ on variables that are random at time $t$. The agent's problem can written recursively as

$$V(x; Z) = \max_{c,d,n,l \geq 0} \log c + \beta E\left\{V(x'; Z')\right\}$$

$$s.t. \tag{19}$$

$$c + d + n = x$$

$$x' = (1 - \tau')\left[(z' - w)l + R^d d + R^{E'} n\right]$$

where $Z = \left[A, \{x^j\}_j\right]'$ is a vector containing aggregate productivity $A$ and the wealth of all agents $j$, and the expectation is taken over the distribution of $z'$ and $Z'$. The variables $w$ and $R^d$ are known when the agent chooses $c$, $d$, $n$, and $l$, but $R^{E'}$ and $\tau'$ depend on the realized values of $Z'$.

We guess and verify that the value function takes the form $V(x; Z) = b \log x + f(Z)$ for some function $f$ and constant $b$. Rewrite the agent's choice variables in terms of $s = d + n$, $n = ys$, and $\phi = l/s$, so that $d = (1 - y)s$ and $\log x' = \log(1 - \tau') + \log \Delta' + \log s$, where $\Delta'$ is defined in equation (9). Then plugging in the guess for the form of the value function, the first-order condition for $s$ yields

$$\frac{1}{x - s} = \beta b E \frac{1}{s}$$

$$\therefore \ s = \frac{\beta b}{1 + \beta b} x$$

from which it follows from the budget constraint that $c = \frac{1}{1+\beta b} x$. This leads directly to the first-order conditions for $\phi$ and $, y$, equations (7) and (8), which are independent of $x$. To

verify the guess, plug the optimal policies and the guess into equation (19) to obtain

$$b \log x + f(z, Z) = \log \left[ (1 - \beta) x \right] + \beta E \left\{ b \log \left( 1 - \tau' \right) + b \log \Delta' + b \log \left[ \beta x \right] + f \left( z', Z' \right) \right\}$$

$$= \log x + \beta b \log x + \text{terms independent of x}$$

so that $b = 1 + \beta b$ and therefore $b = \frac{1}{1-\beta}$, which verifies the guess and completes the proof.

## Proof of Proposition 2

First suppose that $z_{t+1}^i$ is a known constant for all $t$. If $z$ is not random, then equation (31) reduces to $z = w$, and agents are completely indifferent to any level of $\phi_t$. Total labor is then pinned down by equation (12) in order to make the wage equal the value of $z$.

Wealth evolution becomes

$$x_{t+1}^i = (1 - \tau_{t+1}) \left[ y R_{t+1}^E + (1 - y_t) R_t^d \right] \beta x_t^i. \tag{20}$$

Since $\lambda = 0$, equation (5) can be rearranged to yield

$$1 - \tau_{t+1} = 1 - \frac{\int \left\{ R_t^d d_t - A_{t+1} \varepsilon k_t^\alpha \right\}^+ dF(\varepsilon)}{\left[ y_t R_{t+1}^E + (1 - y_t) R_t^d \right] \beta \int x_t^i di} \tag{21}$$

$$\therefore (1 - \tau) \left[ y_t R_{t+1}^E + (1 - y_t) R_t^d \right] = y_t R_{t+1}^E + (1 - y_t) R_t^d - \frac{1}{\beta X_t} \int \left\{ R_t^d d_t - A_{t+1} \varepsilon k_t^\alpha \right\}^+ dF(\varepsilon)$$

$$= y_t R_{t+1}^E + (1 - y_t) R_t^d - R^d (1 - y) F \left( \underline{\varepsilon}_{t+1} \right)$$

$$+ A_{t+1} (\beta X_t)^{\alpha-1} \int_{-\infty}^{\underline{\varepsilon}_{t+1}} \varepsilon dF(\varepsilon).$$

where $\underline{\varepsilon}_{t+1}$ is defined in equation (4) and we have used the fact that in the aggregate $k_t =$

$s_t = \beta X_t$ and $d_t = (1 - y_t) \beta X_t$. Meanwhile, we have using equation (3) that

$$
\begin{aligned}
y_t R_{t+1}^E + (1 - y_t) R_t^d &= A_{t+1} (\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} e^{\sigma\varepsilon} dF(\varepsilon) - \left[1 - F\left(\underline{\varepsilon}_{t+1}\right)\right] (1 - y_t) R_t^d + (1 - y_t) R_t^d \\
&= A_{t+1} (\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} e^{\sigma\varepsilon} dF(\varepsilon) + F\left(\underline{\varepsilon}_{t+1}\right) (1 - y_t) R_t^d,
\end{aligned}
\tag{22}
$$

which plugging into equation (21) yields

$$
(1 - \tau) \left[y_t R_{t+1}^E + (1 - y_t) R_t^d\right] = A_{t+1} (\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} \varepsilon dF(\varepsilon)
$$

so that equation (20) becomes

$$
\begin{aligned}
x_{t+1}^i &= \left[A_{t+1} (\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} \varepsilon dF(\varepsilon)\right] \beta x_t^i \\
&= A_{t+1} (\beta X_t)^{\alpha-1} \beta x_t^i,
\end{aligned}
$$

where the second line uses the fact that $E\{\varepsilon\} = 1$.

This implies that both individual and aggregate wealth growth, and through it the dynamics of consumption and welfare, are unaffected by any policy that changes $y$ or $\underline{\varepsilon}_{t+1}$. Thus a capital requirement that changes the return on the agents' wealth by affecting $\underline{\varepsilon}_{t+1}$ (equation 22) has an exactly offsetting effect on taxes levied by the government to repay depositors at failed banks.

Now suppose that $z_{t+1}^i$ is random. In this case, wealth evolution is given by

$$
\begin{aligned}
x_{t+1}^i &= (1 - \tau_{t+1}) \left[\left(z_{t+1}^i - w_t\right) \phi_t + y R_{t+1}^E + (1 - y_t) R_t^d\right] \beta x_t^i. \\
&= \left[\left(z_{t+1}^i - w_t\right) \phi_t + A_{t+1} (\beta X_t)^{\alpha-1}\right] \beta x_t^i
\end{aligned}
\tag{23}
$$

where the last line plugs in equations (3) and (5) after including the agent's idiosyncratic project income $\left(z_{t+1}^i - w_t\right) \phi_t$. It is apparent from equation (23) that equity investment $y_t$

and the default threshold $\underline{\varepsilon}_{t+1}$ do not affect the growth rate of wealth directly. Instead, we will show that increasing $\zeta$ when the capital requirement binds reduces steady-state $X_t$ and the growth rates of individual wealth.

To show this, by equation (23) it suffices to show that the effect of increasing $\zeta$ is to reduce $\left( z_{t+1}^i - w_t \right) \phi_t$. This immediately reduces the growth rate of $x_t^i$, since individual agents take $X_t$ as given, but it also reduces steady-state $X_t$ because $\alpha < 1$; this can be seen by integrating equation (23) over $i$ and solving for steady-state $X$.

The rest of the proof proceeds as follows: first, we show that increasing $\zeta$ reduces $\underline{\varepsilon}_{t+1}$ when the capital requirement binds; then, we show that reducing $\underline{\varepsilon}_{t+1}$ reduces agent's labor project income $\left( z_{t+1}^i - w_t \right) \phi_t$.

Now because $A_{t+1}$ is a constant, it must be that $R_{t+1}^E$ and $R_t^d$ are also constants; and from equation (8) it must be that $R^E = R^d$. Thus plugging equation (4) into equation (22) and rearranging yields

$$
yR^E + (1-y) R^d = R = \frac{A_{t+1} \underline{\varepsilon}_{t+1} \left( \beta X_t \right)^{\alpha-1}}{1-\zeta}
$$

$$
= A_{t+1} \left( \beta X_t \right)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} \varepsilon dF \left( \varepsilon \right) + F \left( \underline{\varepsilon}_{t+1} \right) A_{t+1} \underline{\varepsilon}_{t+1} \left( \beta X_t \right)^{\alpha-1}
$$

$$
\therefore \quad \frac{\underline{\varepsilon}_{t+1}}{1-\zeta} = \int_{\underline{\varepsilon}_{t+1}}^{\infty} \varepsilon dF \left( \varepsilon \right) + F \left( \underline{\varepsilon}_{t+1} \right) \underline{\varepsilon}_{t+1} \tag{24}
$$

where we have assumed that $y_t = \zeta$ binds. Totally differentiating equation (24) yields

$$
\frac{1}{1-\zeta} d\underline{\varepsilon}_{t+1} + \frac{\underline{\varepsilon}_{t+1}}{(1-\zeta)^2} d\zeta = -\underline{\varepsilon}_{t+1} f \left( \underline{\varepsilon}_{t+1} \right) d\underline{\varepsilon}_{t+1} + f \left( \underline{\varepsilon}_{t+1} \right) \underline{\varepsilon}_{t+1} d\underline{\varepsilon}_{t+1} + F \left( \underline{\varepsilon}_{t+1} \right) d\underline{\varepsilon}_{t+1}
$$

$$
= F \left( \underline{\varepsilon}_{t+1} \right) d\underline{\varepsilon}_{t+1}
$$

$$
\therefore \quad \frac{d\underline{\varepsilon}_{t+1}}{d\zeta} = \underline{\varepsilon}_{t+1} \left[ (1-\zeta)^2 \left( F \left( \underline{\varepsilon}_{t+1} \right) - \frac{1}{1-\zeta} \right) \right]^{-1} \leq 0
$$

where $f \left( \cdot \right) \equiv F' \left( \cdot \right)$ is the pdf of $\varepsilon$, and the last line must be less than zero since the range of $F$ is $[0,1]$ because it is a CDF, $\zeta < 1$, and $\underline{\varepsilon}_{t+1} \geq 0$ since the support of $\varepsilon$ is $[0, \infty)$. Thus

increasing the capital requirement $\zeta$ weakly reduces the default threshold $\underline{\varepsilon}_{t+1}$.

Now rewrite equation (22) as

$$
\begin{aligned}
y_t R_{t+1}^E + (1 - y_t) R_t^d &= A_{t+1} (\beta X_t)^{\alpha-1} \int_{\underline{\varepsilon}_{t+1}}^{\infty} \varepsilon dF(\varepsilon) + F(\underline{\varepsilon}_{t+1})(1 - y_t) R_t^d \\
&= A_{t+1} (\beta X_t)^{\alpha-1} \int_{-\infty}^{\infty} \max\{\varepsilon, \underline{\varepsilon}_{t+1}\} dF(\varepsilon)
\end{aligned}
$$

which is clearly an increasing function of $\underline{\varepsilon}_{t+1}$. Thus increasing $\zeta$ not only lowers $\underline{\varepsilon}_{t+1}$, but this also lowers the agent's portfolio return $yR^E + (1 - y) R^d$.

Because $R^E = R^d$, we can rewrite equation (7) as

$$
0 = E\left\{ \frac{z' - w}{(z' - w)\phi + R^d} \right\}, \tag{25}
$$

which must have a unique solution for $\phi$ because the objective is globally concave. Thus if $\phi$ solves equation (25), it must be that $R^d$ and $\phi$ move in the same proportion (holding $w$ fixed). Thus a decrease in $R^d$ is a reduction in the labor-demand curve of agents (see the lower panel of Figure 1). Because the labor-supply curve (12) is upward-sloping, $w$ also falls in equilibrium but nevertheless, it must be that agents choose a lower value of $\phi_t$ at the higher $\zeta$.

As before, because $\lambda = 0$ the reduction in the returns to bank equity and deposits is exactly offset by a reduction in $\tau_{t+1}$; however, there is no force that offsets the reduction in agents' labor demand.

## Proof of Proposition 3

The result that welfare does not change without idiosyncratic shocks follows as a corollary of Proposition 2. In this case, capital requirements have no real effects, and thus welfare is unchanged.

With idiosyncratic shock, recall that our measure of welfare of entrepreneurs is defined

as $V(X_t)$, where $X_t$ is the steady-state value of average wealth. From Proposition 2, $X_t$ decreases in response to an increase in $\zeta$ when the capital requirement constraint is binding. Since $V(\cdot)$ is strictly increasing as shown in the proof of Proposition 1, welfare of entrepreneurs decreases.

Next, consider laborers. Their welfare is given by (6). Plugging their budget constraint and the first-order condition in (12) into the objective function in (6), and rearranging, we obtain that their welfare is

$$\nu_1 \frac{l^{1+\frac{1}{\nu_2}}}{1 + \nu_2}$$

which is increasing in $l$. As shown in Proposition 2, an increase in $\zeta$ and a binding capital requirement implies a reduction in $\phi_t$ and $X_t$, and, from Proposition 1, of $l_t$, so that the welfare of laborers decreases too.

# B   Solution Method

## B.1   Continuous-Shock Model

In this section we replace $\varepsilon$ in all formulas with the lognormal reparameterization $e^{\sigma\varepsilon}$, where $\varepsilon \sim \mathbb{N}\{0,1\}$.[8] We also replace the banker's problem (2) with (16). Thus, equation (4) becomes

$$R_t^d d_t = \left[ A_{t+1} k_t^\alpha + (1-\delta) k_t \right] e^{\sigma \underline{\varepsilon}_{t+1}}$$

$$\therefore \quad R_t^d = \frac{\left[ A_{t+1} (\beta X_t)^{\alpha-1} + 1 - \delta \right] e^{\sigma \underline{\varepsilon}_{t+1}}}{1 - y_t} \tag{26}$$

$$\therefore \quad \sigma \underline{\varepsilon}_{t+1} = \log R_t^d - \log \left[ A_{t+1} (\beta X_t)^{\alpha-1} + 1 - \delta \right] + \log (1 - y_t)$$

---

[8]Alternatively, we could assume that $\varepsilon \sim \mathbb{N}\left\{-\frac{1}{2}\sigma^2, \sigma^2\right\}$, and replace $\varepsilon$ with $e^\varepsilon$. We find the notation in the text to be the clearest exposition, since in that case $\Phi\{\cdot\}$ is the standard normal CDF. However, this notation does result in $e^{\frac{1}{2}\sigma^2}$ terms popping up in several places.

where the second line uses the fact that in equilibrium $k_t = d_t + n_t = s_t = \beta X_t$ and $d_t = (1 - y_t) s_t = (1 - y_t) \beta X_t$. Equation (26) defines a relation between $\underline{\varepsilon}_{t+1}$ and the realized value of $A_{t+1}$; when $A_{t+1}$ is higher, $\underline{\varepsilon}_{t+1}$ is lower (fewer banks default). The values of $X_t$, $R_t^d$, and $y_t$ are set at $t$, before $A_{t+1}$ and $\underline{\varepsilon}_{t+1}$ are realized; thus taking differences of the last line of equation (26) for any two values of $A_{t+1}$, say $A^*$ and $A$, and rearranging yields

$$\underline{\varepsilon}_{t+1} = \underline{\varepsilon}_{t+1}^* - \frac{1}{\sigma} \left( \log \left[ A (\beta X_t)^{\alpha-1} + 1 - \delta \right] - \log \left[ A^* (\beta X_t)^{\alpha-1} + 1 - \delta \right] \right). \tag{27}$$

Thus to solve for an equilibrium (given aggregate wealth $X_t$), we need only solve for the single value $\underline{\varepsilon}_{t+1}$ that solves equation (26) when $A_{t+1} = A^*$; the other default thresholds then follow from equation (27).

In what follows we assume that $\varepsilon \sim \mathbb{N}(0, 1)$. Then using the fact that

$$\int_a^b e^{\sigma \varepsilon} d\Phi\{\varepsilon\} = e^{\frac{1}{2}\sigma^2} \left[ \Phi\{b - \sigma\} - \Phi\{a - \sigma\} \right]$$

where $\Phi\{\cdot\}$ is the standard normal CDF, we have that equation (3) becomes

$$
\begin{aligned}
R_{t+1}^E &= \frac{\int_{\underline{\varepsilon}_{t+1}}^{\infty} \left[ (A_{t+1} k_t^\alpha + (1-\delta) k_t) e^{\sigma \varepsilon} - R_t^d d_t \right] d\Phi\{\varepsilon\}}{n_t} \\
&= \frac{\left[ A_{t+1} (\beta X_t)^\alpha + (1-\delta) \beta X_t \right] e^{\frac{1}{2}\sigma^2} \left( 1 - \Phi\{\underline{\varepsilon}_{t+1} - \sigma\} \right) - R_t^d (1 - y_t) \beta X_t \left( 1 - \Phi\{\underline{\varepsilon}_{t+1}\} \right)}{y_t \beta X_t} \\
&= \frac{1}{y_t} \left[ \left( A_{t+1} (\beta X_t)^{\alpha-1} + 1 - \delta \right) e^{\frac{1}{2}\sigma^2} \left( 1 - \Phi\{\underline{\varepsilon}_{t+1} - \sigma\} \right) - R_t^d (1 - y_t) \left( 1 - \Phi\{\underline{\varepsilon}_{t+1}\} \right) \right]
\end{aligned}
$$

so that

$$
\begin{aligned}
y_t R_{t+1}^E + (1 - y_t) R_t^d &= \left( A_{t+1} (\beta X_t)^{\alpha-1} + 1 - \delta \right) e^{\frac{1}{2}\sigma^2} \left( 1 - \Phi\{\underline{\varepsilon}_{t+1} - \sigma\} \right) \\
&\quad + (1 - y_t) R_t^d \Phi\{\underline{\varepsilon}_{t+1}\}
\end{aligned} \tag{28}
$$

and

$$R_{t+1}^E - R_t^d = \frac{1}{y_t}\left[\left(A_{t+1}\left(\beta X_t\right)^{\alpha-1} + 1 - \delta\right)e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right)\right.$$
$$\left. - R_t^d\left(1 - (1 - y_t)\,\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right]. \tag{29}$$

Plugging equations (26), (28) and (29) into the agents' first-order conditions (7) and (8) yields

$$0 = E_t\left\{\frac{\left(A_{t+1}\left(\beta X_t\right)^{\alpha-1} + 1 - \delta\right)\left(e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right) - \frac{1 - (1 - y_t)\Phi\left\{\underline{\varepsilon}_{t+1}\right\}}{1 - y_t}e^{\sigma\underline{\varepsilon}_{t+1}}\right)}{\left(z_{t+1} - w_t\right)\phi_t + \left(A_{t+1}\left(\beta X_t\right)^{\alpha-1} + 1 - \delta\right)\left[e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right) + e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]}\right\} \tag{30}$$

and

$$0 = E_t\left\{\frac{z_{t+1} - w_t}{\left(z_{t+1} - w_t\right)\phi_t + \left(A_{t+1}\left(\beta X_t\right)^{\alpha-1} + 1 - \delta\right)\left[e^{\frac{1}{2}\sigma^2}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right) + e^{\sigma\underline{\varepsilon}_{t+1}}\Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right]}\right\}. \tag{31}$$

The tax rate $\tau_{t+1}$ from equation (5) becomes

$$
\tau_{t+1} = \frac{\int_{-\infty}^{\varepsilon_{t+1}} \left[ R_t^d d_t - \left( A_{t+1} k_t^\alpha + (1-\delta) k_t \right) e^{\sigma \varepsilon} \right] d\Phi \{\varepsilon\} + \frac{\lambda}{2} \left[ \int_{-\infty}^{\varepsilon_{t+1}} \left( A_{t+1} k_t^\alpha + (1-\delta) k_t \right) e^{\sigma \varepsilon} d\Phi \{\varepsilon\} \right]^2}{\left[ (\bar{z} - w_t) \phi_t + y_t R_{t+1}^e + (1 - y_t) R_t^d \right] s_t}
$$

$$
= \frac{R_t^d (1 - y_t) \beta X_t \Phi \{\varepsilon_{t+1}\} - \left( A_{t+1} (\beta X_t)^\alpha + (1-\delta) \beta X_t \right) e^{\frac{1}{2}\sigma^2} \Phi \{\varepsilon_{t+1} - \sigma\} + \frac{\lambda}{2} \left[ \left( A_{t+1} (\beta X_t)^\alpha + (1-\delta) \beta X_t \right) e^{\frac{1}{2}\sigma^2} \Phi \{\varepsilon_{t+1} - \sigma\} \right]^2}{\left[ (\bar{z} - w_t) \phi_t + y_t R_{t+1}^e + (1 - y_t) R_t^d \right] \beta X_t} \tag{32}
$$

$$
= \frac{\left( A_{t+1} (\beta X_t)^{\alpha - 1} + 1 - \delta \right) \left( e^{\sigma \varepsilon_{t+1}} \Phi \{\varepsilon_{t+1}\} - e^{\frac{1}{2}\sigma^2} \Phi \{\varepsilon_{t+1} - \sigma\} \right) + \frac{\lambda}{2} \frac{1}{\beta X_t} \left( A_{t+1} (\beta X_t)^\alpha + (1-\delta) \beta X_t \right)^2 \left[ e^{\frac{1}{2}\sigma^2} \Phi \{\varepsilon_{t+1} - \sigma\} \right]^2}{(\bar{z} - w_t) \phi_t + \left( A_{t+1} (\beta X_t)^{\alpha - 1} + 1 - \delta \right) \left[ e^{\frac{1}{2}\sigma^2} \left( 1 - \Phi \{\varepsilon_{t+1} - \sigma\} \right) + e^{\sigma \varepsilon_{t+1}} \Phi \{\varepsilon_{t+1}\} \right]}
$$

where $\bar{z}$ is the average realized value of $z_{t+1}^i$ across agents, so that the denominator in the first line of equation (32) is realized pre-tax wealth, and the last line plugs in equations (26) and (28).

Finally, the banker's first-order condition from solving (2), if the capital constraint does not bind, is given by

$$
0 = E_t \left\{ \left( \alpha A_{t+1} k_t^{\alpha - 1} + 1 - \delta \right) e^{\frac{1}{2}\sigma^2} \left( 1 - \Phi \{\varepsilon_{t+1} - \sigma\} \right) - R_t^d \left( 1 - \Phi \{\varepsilon_{t+1}\} \right) \right\}
$$

$$
\therefore \quad R_t^d = e^{\frac{1}{2}\sigma^2} \frac{\alpha [\beta X_t]^{\alpha - 1} E_t \left\{ A_{t+1} \left( 1 - \Phi \{\varepsilon_{t+1} - \sigma\} \right) \right\} + (1 - \delta) E_t \left\{ 1 - \Phi \{\varepsilon_{t+1}\} \right\}}{E_t \left\{ \left( 1 - \Phi \{\varepsilon_{t+1}\} \right) \right\}} \tag{33}
$$

where the expected value is taken at time $t$ over the distribution of $A_{t+1}$, and the second line uses the fact that in equilibrium $k_t = d_t + n_t = s_t = \beta X_t$. If the capital constraint does bind, then equation (33) no longer applies (it contains a Lagrange multiplier that appears nowhere else) and instead we have that $y_t = \zeta$.

To solve for an equilibrium in this economy, fix a value of aggregate wealth $X_t$ and $\varepsilon_{t+1}^*$.

Then the values of $\underline{\varepsilon}_{t+1}$ in the other $N$ potential $A_{t+1}$ states are pinned down by equation (27). Then so long as the capital requirement does not bind, plug equation (33) into equation (26) to get

$$1 - y_t = \frac{e^{\sigma\underline{\varepsilon}_{t+1} - \frac{1}{2}\sigma^2}\left(\alpha A_{t+1}\left(\beta X_t\right)^{\alpha-1} + 1 - \delta\right)E_t\left\{\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right)\right\}}{\alpha\left[\beta X_t\right]^{\alpha-1}E_t\left\{A_{t+1}\left(1 - \Phi\left\{\underline{\varepsilon}_{t+1} - \sigma\right\}\right)\right\} + (1-\delta)E_t\left\{1 - \Phi\left\{\underline{\varepsilon}_{t+1}\right\}\right\}} \tag{34}$$

from which we can compute $y_t$. We then solve equation (31) for $\phi_t$; given this value, we verify that the first-order condition (30) holds. We search over $\underline{\varepsilon}_{t+1}^*$ on a grid from $-4$ to $4$ until all three equilibrium conditions (26), (30), and (31) hold.

If the capital constraint binds, that is the $y_t$ we compute from this procedure is lower than $\zeta$, then we fix $y_t = \zeta$ rather than using equation (34). Then we proceed as before, solving equation (31) for $\phi_t$ and then verifying that the first-order condition (30) holds. We can repeat this entire procedure for various values of $X_t$, computing the implied tax rate $\tau_{t+1}$ from equation (32) and checking whether the assumed $X_t$ is a steady-state.

## B.2  Large-Shock Model

Assuming that banks that receive the low value of $\varepsilon$ default, equation (3) becomes

$$y_t R_{t+1}^E = (1 - p_{t+1})\left[A_{t+1}k_t^{\alpha-1}\frac{1 - p_{t+1}\gamma}{1 - p_{t+1}} - (1 - y_t)R_t^d\right]$$

so that the agents' portfolio return is

$$y_t R_{t+1}^E + (1 - y_t)R_t^d = A_{t+1}k_t^{\alpha-1}(1 - p_{t+1}\gamma) + (1 - y_t)R_t^d\left[1 - (1 - p_{t+1})\right]$$

$$= A_{t+1}k_t^{\alpha-1}(1 - p_{t+1}\gamma) + p_{t+1}(1 - y_t)R_t^d \tag{35}$$

which yields equation (18). The excess return on equity can be written as

$$\frac{1}{y_t}\left[y_t R_{t+1}^E - y_t R_t^d\right] = \frac{1}{y_t}\left[A_{t+1}k_t^{\alpha-1}\left(1-p_{t+1}\gamma\right) - \left(1-p_{t+1}\right)\left(1-y_t\right)R_t^d - y_t R_t^d\right]$$

$$= \frac{1}{y_t}\left[A_{t+1}k_t^{\alpha-1}\left(1-p_{t+1}\gamma\right) - \left[1-p_{t+1}\left(1-y_t\right)\right]R_t^d\right]. \tag{36}$$

The numerator in equation (5) is given by

$$T_{t+1} = p_{t+1}\left[(1-y_t)R_t^d - \gamma A_{t+1}\right]\beta X_t + \frac{\lambda}{2}\left(p_{t+1}\left[(1-y_t)R_t^d - \gamma A_{t+1}\right]\beta X_t\right)^2$$

where we have plugged in that $k_t = \beta X_t$.

To solve this model for a given value of aggregate wealth $X_t$, we solve for $R_t^d$ assuming the capital requirement doesn't bind using the bank's first-order condition (13):

$$R_t^d = \alpha A\left(\beta X_t\right)^{\alpha-1}E\left\{\varepsilon|\varepsilon > \gamma\right\}$$

$$= \alpha A\left(\beta X_t\right)^{\alpha-1}\left(1-p_c + p_c\frac{1-\gamma s}{1-s}\right)$$

Given this $R_t^d$, we search over values of $y_t$ in $[0,1]$, plugging in equation (35) into equation (7) and solving for $\phi$ numerically, and then using this value of $\phi$ to evaluate equation (8), plugging in equation (36). We continue untul we find a value of $y_t$ that satisfies equation (8).

If the implied $y_t < \zeta$, then this cannot be an equilibrium because the capital requirement binds. In this case, we know that $y_t = \zeta$, and that $R_t^d$ must be lower than the unconstrained case. We thus search over $R_t^d$, rather than $y_t$, but repeat the same procedure: we solve for $\phi$ numerically using equation (7), using $y_t = \zeta$ and the guessed value of $R_t^d$, and then use the implied value of $\phi$ to evaluate equation (8). We repeat the entire procedure for different values of $X_t$ until we find a steady-state.

However, if in equilibrium it turns out that

$$A\left(\beta X_t\right)^{\alpha-1}\gamma > (1-y_t)R_t^d,$$

then defaulting banks have enough capital so that even when they receive the bad shock, they can still pay their depositors. In this case, banks never default, there are no taxes raised, and no deposit insurance subsidy. In this case we replace $p_{t+1}$ in equations (35) and (36) with zero and continue as described in the previous paragraph.

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